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ABSTRACT: We analyze BPS equations for string-like configurations derived from the M5-brane worldvolume action with a Nambu-Poisson structure constructed in ref. [1, 2]. We solve the BPS equations up to the first order in the parameter g which characterizes the strength of the Nambu-Poisson bracket. We compare our solutions to previously constructed BPS string solitons in the conventional description of M5-brane in a constant three-form background via Seiberg-Witten map, and find agreement.

Contents

1 Introduction

M-theory [3] has been a powerful guide in the study of non-perturbative aspects of string theory. However, its microscopic formulation is still lacking. M-theory branes are important building blocks of M-theory and deeper understanding of them will be crucial for making progress regarding this issue.

Recently a model for multiple M-theory membranes based on Lie 3-algebra proposed in ref. [4–6] has been intensively studied. The model has several promising features for a correct description of multiple M-theory membranes at low energy. On the other hand, its relations to the space-time covariant formulation of M-theory branes are not fully clarified yet.[1] In ref. [1, 2] a new M5-brane action was constructed from the multiple membrane action by choosing a Nambu-Poisson algebra as Lie 3-algebra. It was proposed that this new action may be mapped to more conventional ("ordinary" in the following) description of M5-brane [11–16] in a constant three-form background,[2] in analogy with D-branes in a constant B-field background which has non-commutative and commutative descriptions related via Seiberg-Witten map [17].

In this paper we study BPS string solitons in the M5-brane worldvolume with a Nambu-Poisson structure. These configurations describe an M2-brane ending on an M5-brane. Such BPS string solitons were first constructed in ref. [18] in the conventional description of M5-brane, and they were generalized to the case with a constant three-form background in ref. [19, 20]. From M2-brane worldvolume action, this type of configurations with a constant three-form field background has been studied in ref. [21, 22]. More recently, it

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[1]See [7–10] for investigations on this issue.
[2]Note that space-time covariance is broken only by fixing the three-form background in this case.

was studied from the multiple membrane action in ref. [23] through the deformed Basu-Harvey equation [24], and their work may be complementary to present work. We solve the BPS equations in the first order in the parameter g which characterizes the strength of the Nambu-Poisson bracket. We compare our solutions with the previously constructed BPS string solitons in the ordinary description of M5-brane in constant three-form flux via the Seiberg-Witten map [2, 17], and find nice agreement.

2 String solitons in the M5-brane worldvolume with a Nambu-Poisson structure

2.1 Supersymmetry transformation in the M5-brane action

In this subsection we review the supersymmetry transformation in the M5-brane worldvolume action with a Nambu-Poisson structure constructed in ref. [1, 2] to fix our notation and prepare for the study of BPS equations in the subsequent subsections. The detail of the construction of the M5-brane action can be found in ref. [2]. We will follow the notation of ref. [2] except that we omit "'" from the six dimensional variables and some obvious modifications in the numbering of coordinates. In this model the supersymmetry transformation of the fermionic field Ψ is given as follows:

$$
\begin{aligned}
\delta\Psi &= \mathcal{D}_\mu X^i \Gamma^\mu \Gamma^i \epsilon + \mathcal{D}_{\dot\mu} X^i \Gamma^{\dot\mu} \Gamma^i \epsilon \\
&\quad -\frac{1}{2}\mathcal{H}_{\mu\dot\nu\dot\rho}\Gamma^\mu\Gamma^{\dot\nu\dot\rho}\epsilon - \mathcal{H}_{345}\Gamma_{345}\epsilon \\
&\quad -\frac{g^2}{2}\{X^{\dot\mu},X^i,X^j\}\Gamma^{\dot\mu}\Gamma^{ij}\epsilon + \frac{g^2}{6}\{X^i,X^j,X^k\}\Gamma^{ijk}\Gamma^{345}\epsilon,
\end{aligned}
\tag{2.1}
$$

where fields live on the six dimensional M5-brane worldvolume parametrized by x^μ ($\mu = 0, 1, 2$) and $y^{\dot\mu}$ ($\dot\mu = 3, 4, 5$). X^i's ($i = 6, \cdots, 10$) are scalar fields which describe embedding of the M5-brane in the transverse space. Γ's are eleven dimensional Gamma matrices. The metric on the M5-brane is mostly plus, $\mathrm{diag}(-1, 1, 1, \cdots, 1)$. The fermionic shift symmetry has already been taken into account so that the configuration that all the fields vanish is invariant under the supersymmetry (see section 6 of ref. [2] for more detail). The chirality of the fermion and supersymmetry parameters are chosen as follows:

$$
\Gamma^{012345}\Psi = -\Psi, \qquad \Gamma^{012345}\epsilon = \epsilon.
\tag{2.2}
$$

$\{*, *, *\}$ denotes the Nambu-Poisson bracket which we choose to be the one on $\mathbb{R}^3$:

$$
\{f, g, h\} = \epsilon^{\dot\mu\dot\nu\dot\rho}\frac{\partial f}{\partial y^{\dot\mu}}\frac{\partial g}{\partial y^{\dot\nu}}\frac{\partial h}{\partial y^{\dot\rho}},
\tag{2.3}
$$

where $\epsilon^{\dot\mu\dot\nu\dot\rho}$ is a totally anti-symmetric tensor on $\mathbb{R}^3$ with $\epsilon^{345} = 1$. $X^{\dot\mu}$ is given by

$$
X^{\dot\mu} = \frac{y^{\dot\mu}}{g} + b^{\dot\mu}, \qquad b^{\dot\mu} = \frac{1}{2}\epsilon^{\dot\mu\dot\nu\dot\rho}b_{\dot\nu\dot\rho}.
\tag{2.4}
$$

The covariant derivatives in the directions $\mu = 0, 1, 2$ are given as

$$
\mathcal{D}_\mu\varphi \equiv D_\mu\varphi = \partial_\mu\varphi - g\{b_{\mu\dot\nu}, y^{\dot\nu}, \varphi\},
\tag{2.5}
$$

and those in the directions $\dot\mu = 3, 4, 5$ are given by

$$\mathcal{D}_{\dot\mu}\varphi \equiv \frac{g^2}{2}\epsilon_{\dot\mu\dot\nu\dot\rho}\{X^{\dot\nu}, X^{\dot\rho}, \varphi\}$$

$$= \partial_{\dot\mu}\varphi + g(\partial_{\dot\lambda}b^{\dot\lambda}\partial_{\dot\mu}\varphi - \partial_{\dot\mu}b^{\dot\lambda}\partial_{\dot\lambda}\varphi) + \frac{g^2}{2}\epsilon_{\dot\mu\dot\nu\dot\rho}\{b^{\dot\nu}, b^{\dot\rho}, \varphi\}. \tag{2.6}$$

Here, φ collectively represents "covariant" fields X^i and Ψ. The field strength of the anti-symmetric tensor field is given by

$$\mathcal{H}_{\lambda\dot\mu\dot\nu} = \epsilon_{\dot\mu\dot\nu\dot\lambda}\mathcal{D}_\lambda X^{\dot\lambda}$$

$$= H_{\lambda\dot\mu\dot\nu} - g\epsilon^{\dot\sigma\dot\tau\dot\rho}(\partial_{\dot\sigma}b_{\lambda\dot\tau})\partial_{\dot\rho}b_{\dot\mu\dot\nu}, \tag{2.7}$$

$$\mathcal{H}_{345} = g^2\{X^3, X^4, X^5\} - \frac{1}{g} = \frac{1}{g}(V - 1)$$

$$= H_{345} + \frac{g}{2}(\partial_{\dot\mu}b^{\dot\mu}\partial_{\dot\nu}b^{\dot\nu} - \partial_{\dot\mu}b^{\dot\nu}\partial_{\dot\nu}b^{\dot\mu}) + g^2\{b^3, b^4, b^5\}, \tag{2.8}$$

where V is the "induced volume"

$$V = g^3\{X^3, X^4, X^5\}, \tag{2.9}$$

and H is the linear part of the field strength

$$H_{\lambda\dot\mu\dot\nu} = \partial_\lambda b_{\dot\mu\dot\nu} - \partial_{\dot\mu}b_{\lambda\dot\nu} + \partial_{\dot\nu}b_{\lambda\dot\mu}, \tag{2.10}$$

$$H_{\dot\lambda\dot\mu\dot\nu} = \partial_{\dot\lambda}b_{\dot\mu\dot\nu} + \partial_{\dot\mu}b_{\dot\nu\dot\lambda} + \partial_{\dot\nu}b_{\dot\lambda\dot\mu}. \tag{2.11}$$

2.2 BPS equations for string-like configurations

The type of brane configurations we will study is as follows:

	0	1	2	3	4	5	6	7	8	9	10
M5	◯	◯	◯	◯	◯	◯	−	−	−	−	−
M2 soliton	◯	◯	−	−	−	−	◯	−	−	−	−

where ◯ denotes the direction the brane extends and − denotes the direction the brane localizes. The M2-brane ending on the M5-brane appears as a string in the M5-brane world-volume extending in the 1-st direction. Note that the Nambu-Poisson structure is in the 345 directions. We will study the configurations which preserve half of the supersymmetry parametrized by

$$\Gamma^{016}\epsilon = \mp\epsilon. \tag{2.12}$$

From eq. (2.1) we observe that the supersymmetry transformation parametrized by above ϵ is preserved when the following BPS equations are satisfied:

$$\mathcal{D}_{\hat\mu}X^6 \pm \frac{1}{6}\epsilon_{\hat\mu}{}^{\hat\nu\hat\rho\hat\sigma}\mathcal{H}_{\hat\nu\hat\rho\hat\sigma} = 0, \tag{2.13}$$

and other fields set to zero, where $\hat\mu, \hat\nu = 2, \cdots, 5$. $\epsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma}$ is a totally anti-symmetric tensor with $\epsilon^{2345} = 1$.

2.3 BPS equations and solutions at order g^0

We construct the solutions to the BPS equations (2.13) by expansions in g:

$$
\begin{aligned}
X^6 \equiv \Phi &= \Phi_{(0)} + g\Phi_{(1)} + g^2\Phi_{(2)} + \mathcal{O}(g^3), \\
b_{\mu\dot{\nu}} &= b_{\mu\dot{\nu}(0)} + gb_{\mu\dot{\nu}(1)} + g^2 b_{\mu\dot{\nu}(2)} + \mathcal{O}(g^3), \\
b_{\dot{\mu}\dot{\nu}} &= b_{\dot{\mu}\dot{\nu}(0)} + gb_{\dot{\mu}\dot{\nu}(1)} + g^2 b_{\dot{\mu}\dot{\nu}(2)} + \mathcal{O}(g^3).
\end{aligned}
\tag{2.14}
$$

At order g^0, the BPS equation (2.13) becomes

$$
\partial_{\dot{\mu}}\Phi_{(0)} \pm \frac{1}{6}\epsilon_{\dot{\mu}}{}^{\dot{\nu}\dot{\rho}\dot{\sigma}} H_{\dot{\nu}\dot{\rho}\dot{\sigma}(0)} = 0.
\tag{2.15}
$$

From the condition that $H_{(0)}$ can be written as $H_{(0)} = db_{(0)}$ in an open patch, i.e. from the condition $dH_{(0)} = 0$, we obtain the condition

$$
\Box\Phi_{(0)} = 0,
\tag{2.16}
$$

where $\Box \equiv \delta^{\dot{\mu}\dot{\nu}}\partial_{\dot{\mu}}\partial_{\dot{\nu}}$. We consider delta-function source at the origin, like in the case of Dirac monopole, and eq. (2.16) is not satisfied globally. Thus we have

$$
\Phi_{(0)} = \frac{m}{r^2},
\tag{2.17}
$$

where $r^2 = \sum_{\dot{\mu}=2}^{5}(x_{\dot{\mu}})^2$ and

$$
m = \frac{k}{(2\pi)^{3/2}\sqrt{T_6}},
\tag{2.18}
$$

where the integer k is a topological charge of the solution, and T_6 is the tension of the M5-brane with the Nambu-Poisson structure.[3] Corresponding tensor field configurations are given by

$$
b_{(0)}^{\dot{\mu}} = \mp\frac{mx_{\dot{\mu}}}{a^3}A,
\tag{2.19}
$$

where

$$
A = \pm\frac{\pi}{2} + \tan^{-1}\left(\frac{x_2}{a}\right) + \frac{ax_2}{r^2},
\tag{2.20}
$$

with

$$
a^2 = x_3^2 + x_4^2 + x_5^2.
\tag{2.21}
$$

The solution for the tensor field has been studied in ref. [25]. Note that we have chosen the gauge $b_{23} = b_{24} = b_{25} = 0$ which simplifies our analysis. The choice of $\pm$ in (2.20) corresponds to the choice of the direction of the Dirac string. At order g^0 the Dirac string is not physical. The g expansion is not a good expansion for studying the fate of the Dirac string, since g is associated with the Nambu-Poisson bracket which has three derivatives,

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[3] We follow the notation of ref. [2]. See section 7 of the reference for the Dirac quantization condition.

	$y^{\dot\mu}$	X^i $(\Phi \equiv X^6)$	$b_{\mu\dot\nu},\, b_{\dot\mu\dot\nu}$	g	T_6
mass dimension	-1	-1	-1	0	6

Table 1. Mass dimension of the relevant fields and parameters.

and it follows that it is actually the expansion in $g/(\sqrt{T_6}\,a^3)$. One can deduce it from the mass dimension counting and the explicit form of the zero-th order solution. Table 1 summarizes the mass dimension of the relevant fields and parameters in our convention for readers' convenience. Such expansion is not appropriate for $a^3 \lesssim (g/\sqrt{T_6})$. Therefore, in the rest of the paper we are satisfied with that the Dirac string is a gauge artifact at order g^0 and do not worry too much about the Dirac string. In the case of monopoles in non-commutative space, it has been shown that the Dirac string becomes physical and smooth due to the effect non-perturbative in the non-commutative parameter [26].

2.4 BPS equations and solutions at order g

Now we move on to the order g solutions. We should solve

$$0 = \partial_2 \Phi_{(1)} \pm \left(H_{345(1)} + \frac{1}{2}(\partial_{\dot\mu} b_{(0)}^{\dot\mu} \partial_{\dot\nu} b_{(0)}^{\dot\nu} - \partial_{\dot\mu} b_{(0)}^{\dot\nu} \partial_{\dot\nu} b_{(0)}^{\dot\mu}) \right), \tag{2.22}$$

$$0 = \partial_{\dot\lambda} \Phi_{(1)} + (\partial_{\dot\mu} b_{(0)}^{\dot\mu} \partial_{\dot\lambda} \Phi_{(0)} - \partial_{\dot\lambda} b_{(0)}^{\dot\nu} \partial_{\dot\nu} \Phi_{(0)}) \pm \frac{1}{2} \epsilon_{\dot\lambda}{}^{2\dot\mu\dot\nu} H_{2\dot\mu\dot\nu(1)}. \tag{2.23}$$

As in the previous subsection, we solve the condition $dH_{(1)} = 0$. This condition reduces to

$$\Box \Phi_{(1)} = \pm m^2 \left(\frac{16 x_2}{r^8} + \frac{32 A}{a r^6} \right). \tag{2.24}$$

We found

$$\Phi_{(1)} = \pm m^2 \left(\frac{2 x_2}{r^6} + \frac{2 A}{a r^4} \right), \tag{2.25}$$

solves eq. (2.24) while it does not modify the boundary conditions on Φ at $r \to \infty$. Note that Φ is not gauge invariant. (See ref. [2] for gauge transformation laws in the M5-brane worldvolume action with the Nambu-Poisson structure.)

2.5 Seiberg-Witten map

Seiberg-Witten map was first found as a map between non-commutative description and commutative description of D-branes in a constant B-field background [17]. In ref. [2] it was generalized to a map between description by the M5-brane with the Nambu-Poisson structure and description by the ordinary M5-brane in a constant three-form background. As a first step, we study the Seiberg-Witten map for the scalar field. Only in this subsection, we denote the scalar field in the Nambu-Poisson description as $\hat\Phi$, and the corresponding field in the ordinary description as Φ.

BPS string solitons in M5-brane in constant three-form flux have been constructed in ref. [19, 20]. The scalar configuration $\check{\Phi}$ is given by

$$\check{\Phi} = \frac{m}{\check{r}^2} \pm \tan\theta \, \check{x}_2, \tag{2.26}$$

where θ is related to the background three-form field as $H_{345}^{(bg)} = -\tan\theta$.[4] Here $\check{x}_{\dot{\mu}} = x_{\dot{\mu}}$ ($\dot{\mu} = 3, 4, 5$), but for $\check{x}_2$ we need to make a rotation in the coordinate and the field before applying the Seiberg-Witten map [27–30]:

$$\begin{pmatrix} \Phi \\ x_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \check{\Phi} \\ \check{x}_2 \end{pmatrix}, \tag{2.27}$$

where we choose ϕ so that the term linear in x_2 does not appear in Φ. For $|\theta| \ll 1$, $\phi = \pm\theta + \mathcal{O}(\theta^2)$.

The Seiberg-Witten map for the scalar field is given in ref. [2] up to order g:

$$\hat{\Phi} = \Phi + g b^{\dot{\mu}} \partial_{\dot{\mu}} \Phi + \mathcal{O}(g). \tag{2.28}$$

Up to the first order in g and θ, we obtain

$$\hat{\Phi} = \frac{m}{r^2} \pm 2m^2 \left(\frac{\theta x_2}{r^6} + \frac{gA}{ar^4} \right) + \text{(higher order terms in } g \text{ and } \theta\text{)}. \tag{2.29}$$

This coincides with our solution (2.25) if

$$g = \theta + \mathcal{O}(\theta^2). \tag{2.30}$$

Eq. (2.30) is consistent with the result of ref. [2].

3 Summary and future directions

In this paper we obtained solutions of the BPS equations for string-like configurations derived from the M5-brane worldvolume action with the Nambu-Poisson structure constructed in ref. [1, 2] up to the first order in g. After the Seiberg-Witten map our solutions agreed with the BPS string solitons in the ordinary description of M5-brane. This result motivates more thorough study of Seiberg-Witten map between the M5-brane worldvolume action with the Nambu-Poisson structure and the ordinary M5-brane worldvolume action in constant three-form flux. In the case of D-branes in a constant B-field background, it has been argued (and explicitly checked for the first few terms in the expansion in the slowly varying field strength F of the ordinary gauge field) that the non-commutative $\hat{F}^2$ action coincides with the ordinary DBI action in the zero slope limit, up to total derivative terms and an additive constant [17]. The M5-brane action with the Nambu-Poisson structure, with the $\mathcal{H}^2$ term in it being in parallel with the $\hat{F}^2$ term above, may similarly coincide with

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[4]The solution looks like the one for linearized M5-brane action, but actually it solves the equation of motion of the non-linear M5-brane action in the ordinary description [19, 20]. Our convention differs from that in ref. [19, 20] by a factor of $\frac{1}{4}$.

the ordinary (DBI-type) M5-brane action in an appropriate limit of the M2-brane tension and the background three-form flux. But this needs to be checked by further investigation.[5] To achieve this goal, we first need to understand how to take the appropriate limit. This might not be as simple as in the case of D-branes in a constant B-field background which can be studied using the open string worldsheet free CFT, due to the interacting nature of the membrane worldvolume theory (see ref. [21, 22] for earlier studies). But the investigation through the relation between M-theory and type IIA string theory along the line of ref. [2] may be of help to understand this issue. We also need to understand how to connect the apparently different treatments of the self-dual two-form between the two descriptions of the M5-brane in constant three-form flux.[6]

Our analysis was restricted to the expansion in the parameter g. Such expansion is not suitable for studying the structure near the Dirac string. In the case of solitons/instantons in non-commutative space, techniques to obtain exact solutions have been developed by expressing functions on non-commutative space with operators acting on the Hilbert space of harmonic oscillators [26, 34–44]. In these cases solutions are smooth due to the effect non-perturbative in the non-commutative parameter. To construct solutions on the M5-brane with the Nambu-Poisson structure in a similar way, we would first need to understand what is the appropriate "quantization" of the Nambu-Poisson bracket. For this purpose, investigations in ref. [23, 45] seem very suggestive. It will be very interesting to study solitons on manifolds with a quantum Nambu-Poisson structure from the M-theory point of view.

Acknowledgments

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[5]It would be worthwhile to mention that in the case of D-brane in a constant B-field background, the description by the Poisson bracket may not be just an approximation of the description by the Moyal product, but it can be another description related to commutative or non-commutative descriptions through Seiberg-Witten (type) maps [31, 32]. Similar story may hold in the case of M5-brane in constant three-form flux.

[6]After we added these lines in our draft in response to the referee's comment while we were preparing the final revision of this paper, ref. [33] appeared which made an interesting progress in this direction.

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